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Homotopy Continuation Method of Arbitrary Order of Convergence for Solving the Hyperbolic Form of Kepler's Equation

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Abstract. In this paper, an efficient iterative method of arbitrary integer order of convergence ≥ 2 has been established for solving the hyperbolic form of Kepler's equation. The method is of a dynamic nature in the sense that, moving from one iterative scheme to the subsequent one, only additional instruction is needed. Most importantly, the method does not need any prior knowledge of the initial guess. A property which avoids the critical situations between divergent and very slow convergent solutions that may exist in other numerical methods which depend on initial guess. Computational Package for digital implementation of the method is given and is applied to many case studies.

Key words. Homotopy method—hyperbolic Kepler's equation—initial value problem—orbit determination.

1. Introduction

Many instances of hyperbolic orbits occur in the solar system and recently, among the artificial satellites, lunar and solar probes. Moreover, in some cases of orbit determination for an elliptic orbit, it may very well happen (Escobal 1975) that during the solution process (usually iteration), the eccentricity e becomes greater than unity and the orbit becomes hyperbolic. Also, in the interplanetary transfer, the escape from the departure planet and the capture by the target planet involve hyperbolic orbits (Gurzadyan 1996). On the other hand, in orbit determination of visual binaries provisional hyperbolic orbits are used to represent the periastron section of high-eccentricity orbits of long and indeterminate period (Knudsen 1953). In fact, we should handle hyperbolic orbits frequently when integrating a perturbed motion with the initial condition of nearly parabolic orbits (Fukushima 1997).

From the above, it is then clear that the hyperbolic orbits not only exist naturally, but can also be used to solve some critical orbital situations.

The position–time relation in hyperbolic orbits is known as Kepler's equation for the hyperbolic case and is given as

$$M = e \sinh G - G; \quad 1 \leq e < \infty; \quad 0 \leq M < \infty, \tag{1.1}$$