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Common fixed point results for two new classes of hybrid pairs in symmetric spaces

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ABSTRACT

Some common fixed point theorems due to Abbas and Khan [M. Abbas, A.R. Khan, Common fixed points of generalized contractive hybrid pairs in symmetric spaces, Fixed Point Theor. Appl. 2009 (2009) 11, Article ID 869407, doi:10.1155/2009/869407], and Abbas and Rhoades [M. Abbas, B.E. Rhoades, Common fixed point theorems for hybrid pairs of occasionally weakly compatible mappings defined on symmetric spaces, Pan. Amer. Math. J. 18 (1) (2008) 55–62] are proved for two new classes of hybrid pair of mappings which contain occasionally weakly compatible hybrid pairs as a proper subclass. Consequently, some results proved by Hussain et al. [N. Hussain, M.A. Khamsi, A. Latif, Common fixed points for $\mathcal{JH}$-operators and occasionally weakly biased pairs under relaxed conditions, Nonlinear Anal. 74 (2011) 2133–2140], Bhatt et al. [A. Bhatt, et al., Common fixed point theorems for occasionally weakly compatible mappings under relaxed conditions, Nonlinear Anal. 73 (2010) 176–182] and many others are extended to hybrid pair of mappings. Examples are also presented to support the concepts defined in the paper.

1. Introduction and preliminaries

In 1968, Kannan [22] proved a fixed point theorem for a map satisfying a contractive condition that did not require continuity at each point. This paper was a genesis for a multitude of fixed point papers over the next two decades. Sessa [25] coined the term weakly commuting maps. Jungck [16] generalized the notion of weak commutativity by introducing compatible maps and then weakly compatible maps [17]. Jungck and Rhoades [19] and Al-Thagafi and Shahzad [4] studied fixed point results for occasionally weakly compatible (owc) maps. Abbas and Khan [1] obtained several common fixed point theorems for hybrid pairs of single valued and multivalued owc maps defined on a symmetric space (see also, [2, 3]). Recently, Djorić et al. [10] proved that in the presence of a unique point of coincidence of the given mappings, occasionally weakly compatible maps reduces to the class of weakly compatible maps and therefore no generalization can be obtained in this direction. They pointed out that the case of occasionally weakly hybrid pair of mappings is different and the class of occasionally weakly compatible hybrid pair is larger than the class of weakly compatible hybrid pair of mappings. In this paper, motivated by the work of Hussain et al. [15], we introduce two new and different classes of noncommuting selfmaps. These classes contain the occasionally weakly compatible hybrid pair of mappings as a proper subclass. For these new classes, several fixed point theorems for hybrid pair of single valued and multivalued maps satisfying generalized contractive conditions are established in a symmetric space.

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