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Abstract

A probabilistic approach has been followed to study the uncertainty in predicting uplift pressures under hydraulic structures and the flow field in terms of hydraulic heads The uncertainties considered in this paper include the geological structures and the bedding characteristics. Both horizontal and inclined bedding with different degrees of inclinations have been treated The mean and standard deviation of the hydraulic head field, uplift pressures and total seepage rate under the floor are estimated using Monte-Carlo approach. It has been shown that the uncertainty in predicting the flow field do not depend only upon the contrast in heterogeneity of hydraulic conductivity but upon the bedding inclination as well. The higher the magnitude of the angle of inclination the greater the uncertainty. The maximum uncertainty appeared in case of inclined bedding with 45 degree. Monte-Carlo simulations as used in the current study can be applied in the same way to other particular situations which are not covered in that study.

Introduction

Most studies of seepage under hydraulic structures are based, in large part, on deterministic models of flow through porous media .i.e. the soil properties and geological configurations are assumed to be known with certainty. However in reality, these properties vary from one location to another in space [9] and can only be estimated deterministically through enormous field and laboratory measurements. From practical and economical point of view, it is not feasible to do such enormous measurements and therefore soil properties become uncertain at locations where there are no measurements. A well known but crude method to deal with uncertainty in a system is the use of a safety factor. A more realistic approach is the stochastic modelling. The most general, stratforward and widely used method for stochastic analysis is Monte-Carlo technique. This technique has been followed in the present study to analysis confined steady state seepage problem.

Few attempts have been made to study the effect of variability in soil permeability on confined seepage [4, 6, 7] and unconfined seepage [3]. In such studies the variability in soil permeability is modelled as stationary Gaussian random fields. The main focus in these studies was to establish the relationship between the second moment of seepage characteristics and the second moment of the soil permeability. However, in the present work an attention focused on the influence of geological structure, the shape and inclination of the bedding.

Steady State Seepage problem

The problem studied in this research is confined seepage under hydraulic structures The steady state two-dimensional confined flow is governed by the well known elliptic partial differential equation:

$$\frac{\partial}{\partial x}\left(K_{XX}\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial x}\left(K_{YY}\frac{\partial H}{\partial x}\right)=0 \qquad (1)$$

where, H is the hydraulic head, Kxx, is the hydraulic conductivity in the X-direction, and Kyy is the hydraulic conductivity in the Y-direction.

In this study, the hydraulic conductivity at each point in the domain is assumed to be hydraulically isotropic (i.e. Kxx,= Kyy,= K). However the model presented here is more general that can handle hydraulically anisotropic soils. Because of the interfering of the influences of heterogeneity and point hydraulic anisotropy that may appear in interpretation of the results, a focus is made to study hydraulically isotropic soils.

The aspects which have been focused on in this context are the uplift pressure, the hydraulic head field and the quantity of seepage passing under the foundation. Fig.(1) shows a definition sketch of the problem under study.

The Geological Structures

The analysis in this paper uses the two-dimensional Coupled Markov Chain model developed by Elfeki et al. [1995] to generate several realisations of the geological structures. These realisations are characterised by transition probabilities between the different geological materials or soil types (which are called states in the text) that present in the geological deposit. The Coupled Markov Chain model is fully described by Elfeki [1996]. Three types of geological patterns are considered:

1) Horizontal Bedding. (0.0 degree).

2) Positively Inclined Bedding. (26.5 and 45 degrees).

3) Negatively Inclined Bedding.(-26.5 and -45 degrees).

The definitions of positive inclination and negative inclination are shown in Fig (2). The transition probabilities used to generate the different geological patterns are chosen to be the same to make comparisons possible. The values of these transition probabilities are chosen to represent moderately correlated geological deposit. The transition probabilities are displayed in Table (1). The corresponding output transitions are also calculated from the generated patterns and displayed in the same table to show the reproduction of these transitions in the generated pattern. Good agreement can be observed. Single realisations of the different patterns considered in this study are given in Figs (3) through (7).

Table (1) Input and Output Statistics of Markov Model. from Fig.3 to Fig.7

Length of The Given Section = 100 m
Depth of The Given Section = 20 m
Sampling interval in X-direction = 1 m
Sampling interval in Y-direction = 1 m (Fig. 3,6,7)

= 0.5 m (Fig. 4, 5)

Number of States = 4

Input Statistics to The Model | Calculated Statistics From The Image

Transition Probability Matrix in The Direction of The Bedding

State	1	2	3	4	State	1	2	3	4
1	0.900	0.030	0.030	0.040	1	0.907	0.032	0.026	0.035
2	0.030	0.910	0.030	0.030	2	0.031	0.901	0.038	0.031
3	0.030	0.030	0.920	0.020	3	0.038	0.022	0.930	0.011
4	0.030	0.040	0.030	0.900	4	0.030	0.030	0.015	0.925

Transition Probability Matrix Normal to The Bedding

State	1	2	3	4	State	1	2	3	4
1	0.400	0.200	0.200	0.200	1	0.439	0.166	0.239	0.156
2	0.200	0.400	0.200	0.200	2	0.249	0.348	0.148	0.255
3	0.200	0.200	0.400	0.200	3	0.244	0.168	0.428	0.160
4	0.200	0.200	0.200	0.400	4	0.137	0.161	0.349	0.354

Soil Parameters:

The value of the hydraulic conductivity corresponds to each state (soil type) in Markov model is chosen to represent a sandy deposit. Table (2) shows the soil conductivity that corresponds to each state. The conductivity ranges from course to medium sand.

Table (2) Soil Properties of The Different States.

State	Proportion of The State in Fig.3	Isotropic Hydraulic Conductivity
1	0.257	90 m/day
2	0.189	70 m/day
3	0.267	40 m/day
4	0.197	10 m/day

The contrast in conductivity is displayed in a grey shades scale in top of Figs (3) through (7). Dark areas refer to low conductivity and light areas correspond to high conductivity zones. The spatial average of the conductivity of the generated realisations is estimated to be K_a= 49 m/day and the spatial standard deviation is σ_K=32.5 m/day. The corresponding coefficient of variation is CV= 0.66.

Finite Difference Flow Model

The finite difference method is applied to solve the elliptic equation, Eqn (1), numerically. A sketch of the finite difference mesh used in this study is shown in Fig (1). It contains 2000 grid points in case of discretization 1m x 1m for horizontal bedding and bedding with inclination 45 degree. and contains 4000 grid points in case of discretization 1m x .5 m. for bedding with inclination 26.5 degrees The upstream and downstream head values are fixed at 1 m and 0 m respectively. The Finite difference code for the solution of the governing equation is broadly similar to the one given by Elfeki, 1996 [2]. It is based on the conjugate gradient method [8] in the solution of the system of equations.

Once the system of equations is solved leading to the nodal heads, the output quantities related to the flow rate and uplift pressures are easily deduced. The code has been verified for horizontal floor on homogeneous isotropic soil where analytical solution exists [5].

Stochastic Analysis

The Stochastic analysis followed in this paper is based on Monte-Carlo method. This method works, in brief, as follows. A geological structure is first generated using Markov model with the data displayed in Table (1). The generated image is called a realisation of the subsurface (e.g. the top of Fig. (3)). Then, on that realisation each soil type is assigned an isotropic conductivity value according to its type from Table (2). The seepage problem is then solved under specified boundary conditions (Fig. (1)). The solution leads to the hydraulic head corresponds to that realisation. Another realisation is generated and the flow model is run again, and so on up to the total number of realisations specified by the user. Statistical analysis of the ensemble of the output realisations can be carried out to obtain the mean and the variance of the output variables at each node in the grid. The steady state simulations are carried out on PC SX486 processor with clock speed 33MHz. The computer time of 1000 Monte-Carlo runs is about 10 hours in case of 2000 nodes and for 300 Monte-Carlo runs is about 16 hours in case of 4000 nodes. The total number of runs are 3600. The total time to perform these experiments is about 62 hours.

Single Realisations of The Hydraulic Head Field

Some single realisations of the flow fields in terms of hydraulic heads denoted by dotted lines are displayed in Fig (3) (middle) for horizontal bedding, Fig.(4) (middle) for inclined bedding with 26.5 degree, Fig (5) (middle) for inclined bedding with -26.5 degree, Fig (6) (middle) for inclined bedding with 45 degree, and Fig (7) (middle) for inclined bedding with -45 degree. The contours of hydraulic heads show broken patterns in all figures, due to the heterogeneity in the system. In Fig (3), the contours of the hydraulic heads do not show a symmetric shape along the centre of the concrete floor as in the well known homogeneous case. A positively skewed contour lines are observed in Fig (4) and Fig (6)., while a negatively skewed contour lines are observed in Fig (5) and Fig (7). Although the point conductivity is isotropic, the global behaviour of the flow field that appeared in the form of skewness is anisotropic in nature. This type of anisptropy is accounted for by the influence of the bedding inclination.

Single Realisations of The Uplift Pressure

Fig (8) shows single realisations of the uplift pressure for the various types of bedding. The uplift pressures displayed in this figure correspond to the realisations presented in Figs (3) through (7). The displayed uplifts show different shapes due to the heterogeneity of the deposit and the bedding inclination. One can notice also the comparison with Bligh theory. A general conclusion is that Bligh theory underestimates the uplift pressure in the upstream part of the floor while it overestimates the uplift pressure in the downstream part of the floor. That means a care should be taken in the design of hydraulic structures over heterogeneous soils. The Monte-Carlo method presented in this study can help in that respect This will be explained in the following paragraphs.

Ensemble Statistics of The Hydraulic Head Field

Figs (3) through (7) show the ensemble mean (in the middle of the figures with solid lines) and the ensemble standard deviation of hydraulic head fields (in the bottom of the figures).that calculated over the whole number of realisations for the various geological patterns considered in this paper. The contours of the hydraulic head in these figures show similar pattern to the approach used by Smith & Freeze [6,7] who presented results of numerical experiments on both one- and two-dimensional steady state confined flow problems in auto-correlated random fields. The ensemble fields of the mean and standard deviation of the heads show the same behaviour as the single realisation ones except they are smoother, due to the averaging process. The maximum standard deviations are located under the floor and decrease towards the boundaries where the standard deviations are zero as they considered in this study. This behaviour reflects the nonstationarity in the hydraulic head field due to the specified constant head boundaries of the flow domain.

Ensemble Statistics of The Uplift Pressure

Fig (9) shows the ensemble mean uplift pressure under the concrete floor for the different types of bedding. There is no significant difference between the various types. All cases correspond to a deterministic solution of a homogeneous problem with an average hydraulic conductivity.

Fig (10) displays the ensemble standard deviation of the uplift pressure (the measure of the uncertainty). There is a general trend which shows that the uncertainty increases when the bedding angle differs from zero. The higher the magnitude of the angle of inclination the greater is the uncertainty. Explanation of this behaviour can be made in the light of the correlation pattern (bedding direction), and the flow direction (the head gradient). In case of horizontal bedding the cells are correlated horizontally and the flow direction is horizontally dominant under the floor. This case produces the minimum uncertainty However, in case of inclined bedding the correlation pattern makes an angle with the flow direction. which is still horizontally dominant under the floor. In this case the flow faces more resistance and consequently the head gradients are greater than in case of horizontal bedding. Then on averaging over the realisations the deviation from the mean will be higher that results in greater uncertainty. There is a slight difference in case of -26.5 degree. This may be due to the number of realisations (300 realisations in this case) used are not sufficient to stabilise the variance. However, 1000 realisations are used for horizontal bedding and inclination with 45 degree. The peak of the uncertainty is shifted towards the right (with respect to the floor centre) in case of positively inclined bedding, however, it is shifted to the left in case of negatively inclined bedding comparable with horizontal bedding where the uncertainty looks symmetric. Table (3) shows the maximum uncertainty in the uplift pressure. One may recognise that the peak uncertainty is minimum in case of horizontal bedding, while it increases with bedding angle that differ from zero.

Table 3 Maximum. Uncertainty in The Uplift Pressure.

Type of Bedding	Max. Uncertainty in The Uplift Pressure
Inclined with -45 degree	0.074 m
Inclined with -26.5 degree	0.063 m
Horizontal	0.062 m
Inclined with 26.5 degree	0.091 m
Inclined with 45 degree	0.116 m

Figs (11) through (15) display the ensemble mean uplift pressure (that corresponds to deterministic solution with average conductivity), the uplift pressure of single realisations of Figs.(3) through (7) respectively, the linear theory of Bligh and the 95% confidence intervals of the uplift pressure (that correspond to the mean 2 standard deviation). One can notice that all the single realisations are falling within the bounds of 95% confidence limits. The 95% confidence intervals are wide in case of horizontal bedding and bedding with positive inclination, while the intervals are narrow in case of bedding with negative inclination. It appears that either the use of Bligh theory or the ensemble average solution (that corresponds to a deterministic solution with an average value of the hydraulic conductivity) may lead to results that are significantly different from the result of the actual heterogeneous medium (that represented by the single realisations) The presented stochastic approach seems to give a better consistency by actually accounting for the geological uncertainty. With this approach one would also handle any type of geological setting.

Ensemble Statistics of The Total Quantity of Seepage

The total quantity of seepage passing under the hydraulic structure is calculated at a section in the mid of the floor over the whole depth of the aquifer (20 m). The ensemble average of the seepage rate and the ensemble standard deviation (the measure of the uncertainty) are estimated over the number of Monte-Carlo runs. The calculated values are displayed in Table (4). It can be noticed that the ensemble average quantity of seepage increases with the increase in the bedding angle from negative to positive (from top to bottom in the table). This displays realistic results since the inclination with -45 degree, the maximum negative inclination, is against the flow direction that produce relatively low seepage rate. However, in case of bedding inclination with 45 degree where the flow is more aligned with the bedding inclination. This case produces relatively high seepage rate. The uncertainty does not show general trend. It is about zero at -45 degree inclination and maximum at +45 degree of inclination. The results in between show up and down behaviour. This variation is currently the subject of further investigation by the author.

Table (4) The Ensemble Average and The Uncertainty in The Total Quantity of Seepage.

Type of Bedding	Ensemble Average Seepage	Uncertainty in The Seepage
Inclined with -45 degree	25.41 m3/day/m'	0.000 m3/day/m'
Inclined with -26.5 degree	28.05 m3/day/m'	0.082 m3/day/m'
Horizontal	28.56 m3/day/m	0.035 m3/day/m'
Inclined with 26.5 degree	33.93 m3/day/m'	0.029 m3/day/m'
Inclined with 45 degree	38.23 m3/day/m'	0.116 m3/day/m'

Conclusions

The following concluding remarks can be drawn from this study:

(1) The steady state seepage problem using realistic stochastic descriptions for the configuration of the geological system and the properties of the porous media can provide the designers with confidence limits that account for the uncertainty in the exact configuration of the geological system.

(2) It has been shown that inclined bedding produces global hydraulic anisotropy pattern in the contours of the hydraulic heads although the point hydraulic conductivity is isotropic.

(3) The uncertainty in predicting the flow field does not depend only upon the contrast in heterogeneity of hydraulic conductivity but upon the bedding inclination as well. The higher the magnitude of the angle of inclination the greater the uncertainty. The maximum uncertainty appeared in case of inclined bedding with 45 degrees.

(4) The peak uncertainty in the uplift pressure is located in the centre of the floor in case of horizontal bedding, while, it is shifted towards the right (with respect to the centre of the floor) in case of positively inclined bedding and it is shifted towards the left in case of negative inclination.

(5) It has been shown the limitation of Bligh theory specially in case of inclination with 45 degrees where the uplift pressure is underestimated over about 70% of the floor

length from the downstream end. This may be of some importance in the design of such hydraulic structures over inclined bedding.

(6) It appears that either the use of Bligh theory or the ensemble average solution (that corresponds to a deterministic solution with an average value of the hydraulic conductivity) may lead to results that are significantly different from the result of the actual heterogeneous medium (that represented by the single realisations) The presented stochastic approach seems to give a better consistency by actually accounting for the geological uncertainty.

(7) Monte-Carlo experiments performed in the current study for obtaining numerical solutions to two-dimensional seepage problems in terms of the mean values of the hydraulic heads, uplift pressures, seepage rates and their standard deviations may be taken into account as an example to any other particular situations that are not covered in the present study.

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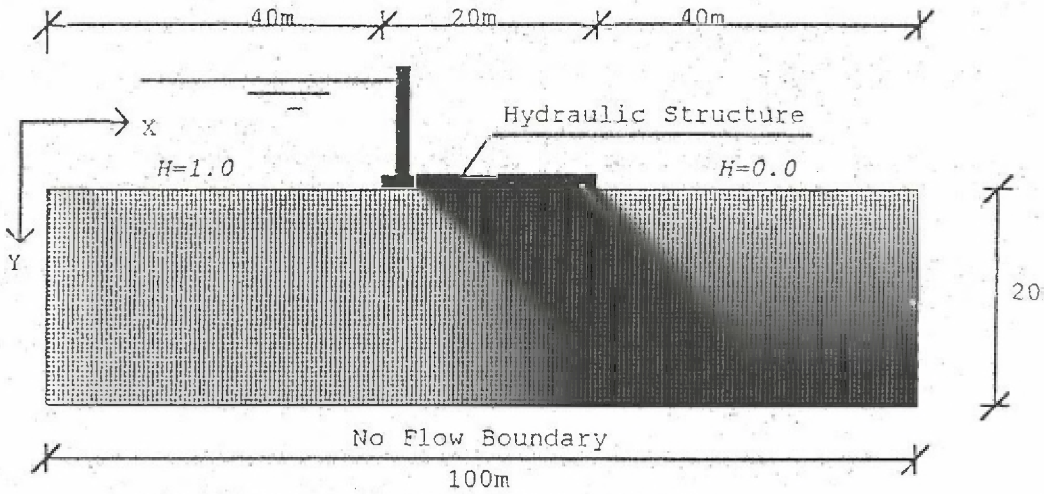

Fig.(1) Sketch of The Seepage Problem.

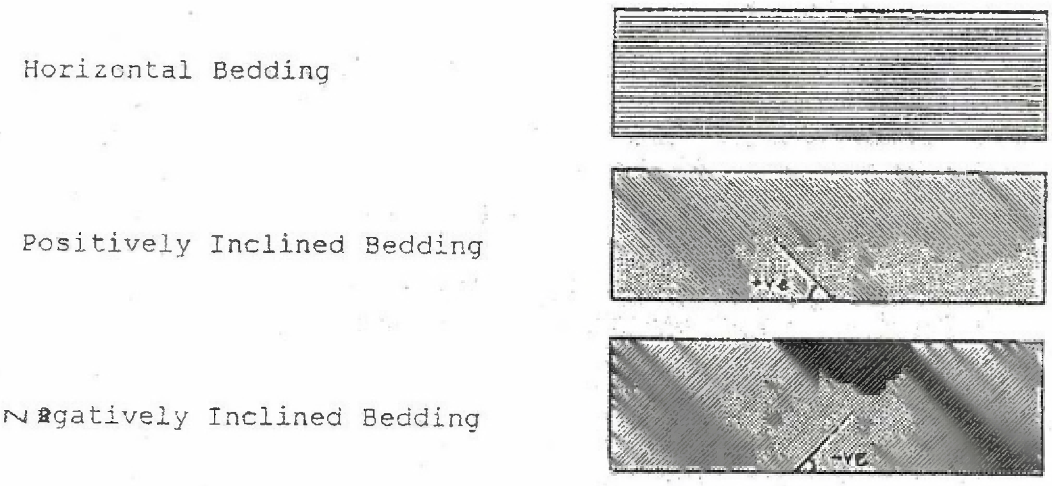

Fig. (2) Definition Sketch of The Different Bedding

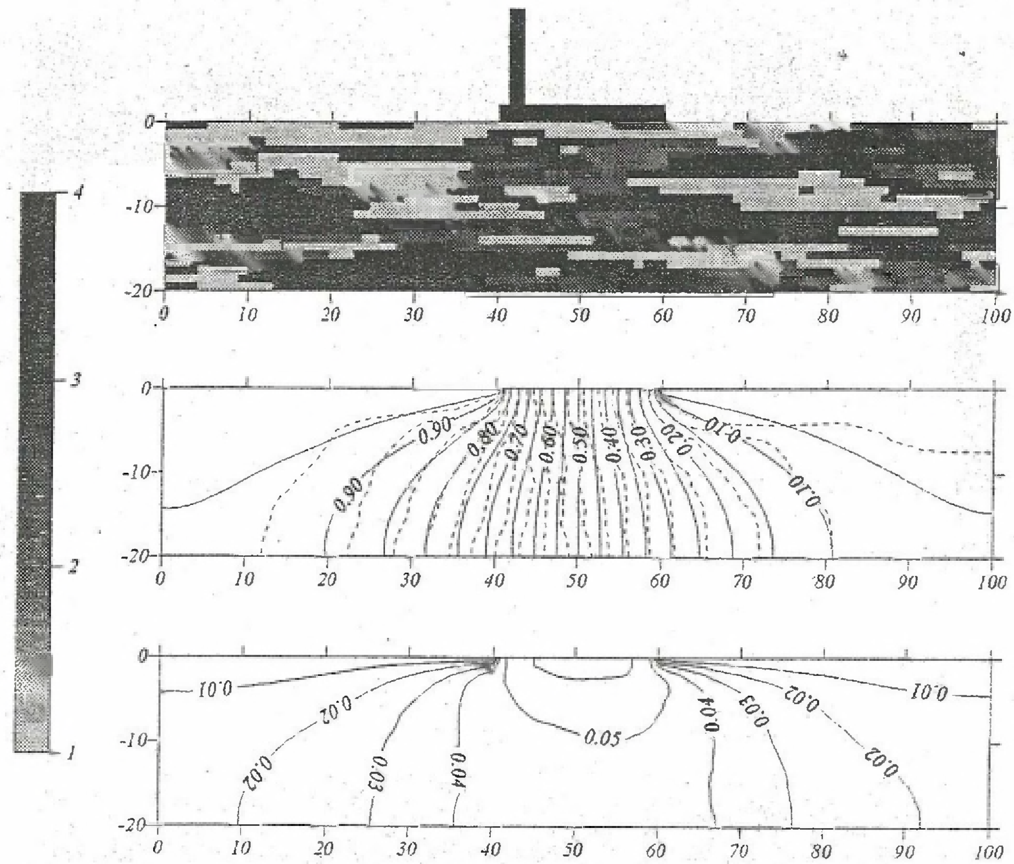

Fig.(3) Single Realisation of The Horizontal Bedding (Top), Ensemble Hydraulic Head Superimposed over The Hydraulic Head of That Realisation (Middle) and Standard Deviation of The Hydraulic Head (Bottom).

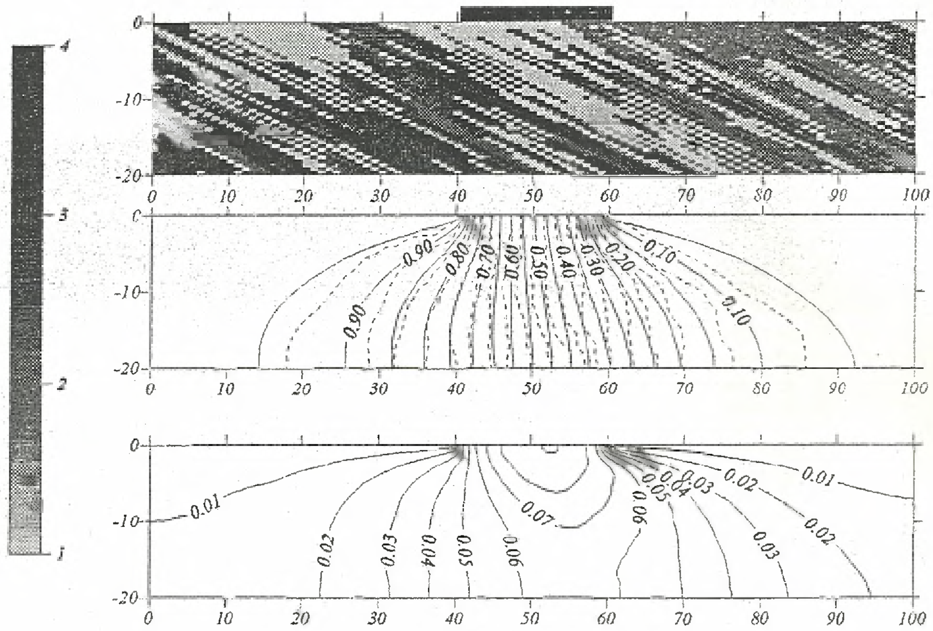

Fig. (4) Single Realisation of Positively Inclined Bedding -26.5 degree (Top), Ensemble Hydraulic Head Superimposed over The Hydraulic Head of That Realisation (Middle) and Standard Deviation of The Hydraulic Head (Bottom).

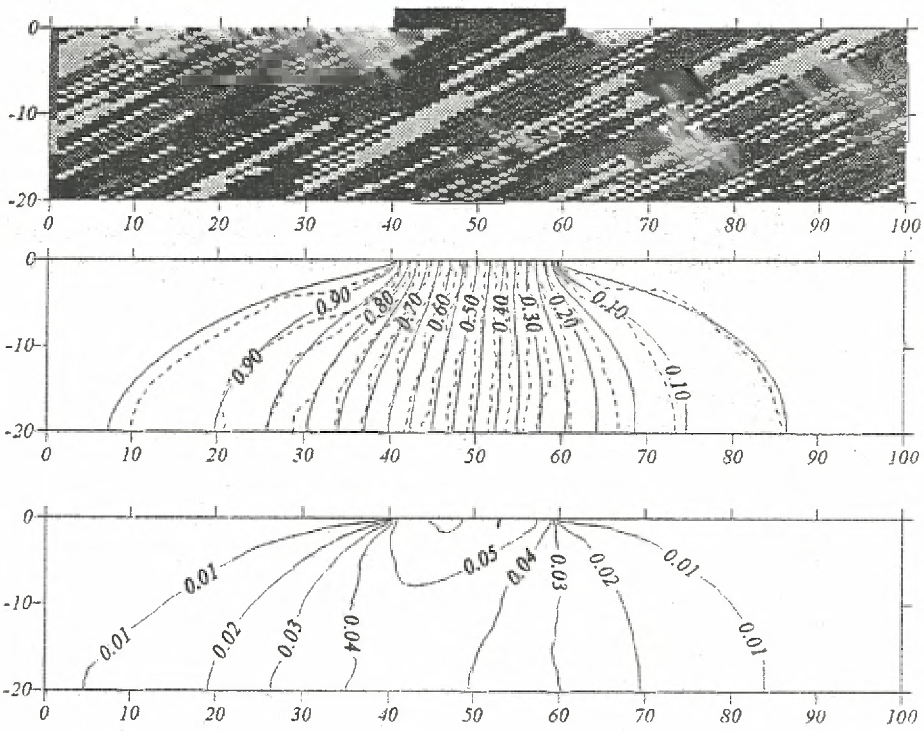

Fig. (5) Single Realisation of Negatively Inclined Bedding -26.5 degree (Top), Ensemble Hydraulic Head Superimposed over The Hydraulic Head of That Realisation (Middle) and Standard Deviation of The Hydraulic Head (Bottom).

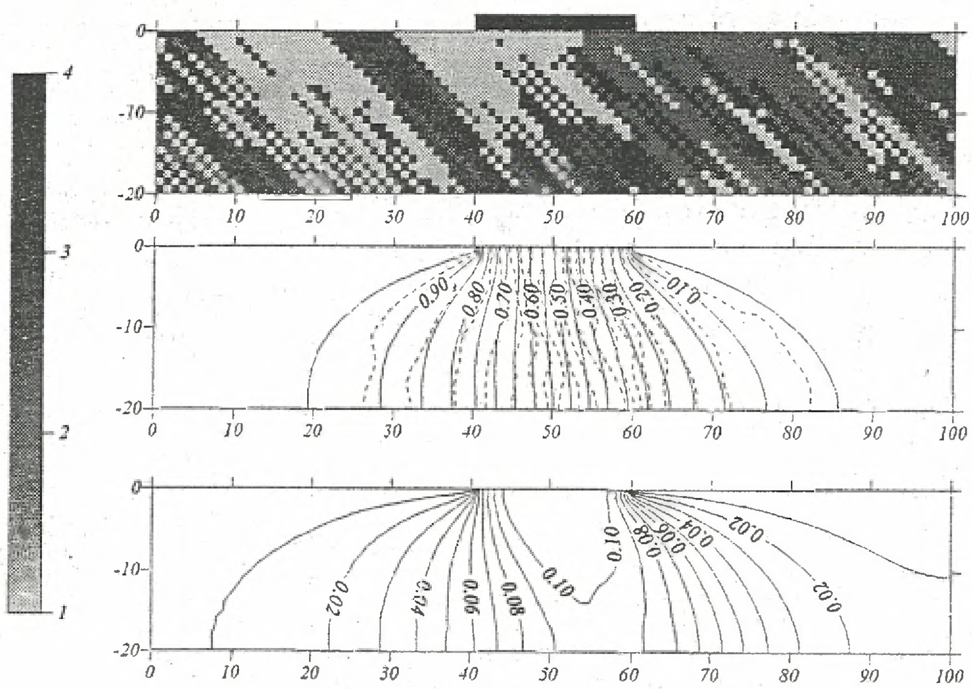

Fig.(6) Single Realisation of Positively Inclined Bedding 45 degree (Top), Ensemble Hydraulic Head Superimposed over The Hydraulic Head of That Realisation (Middle) and Standard Deviation of The Hydraulic Head (Bottom).

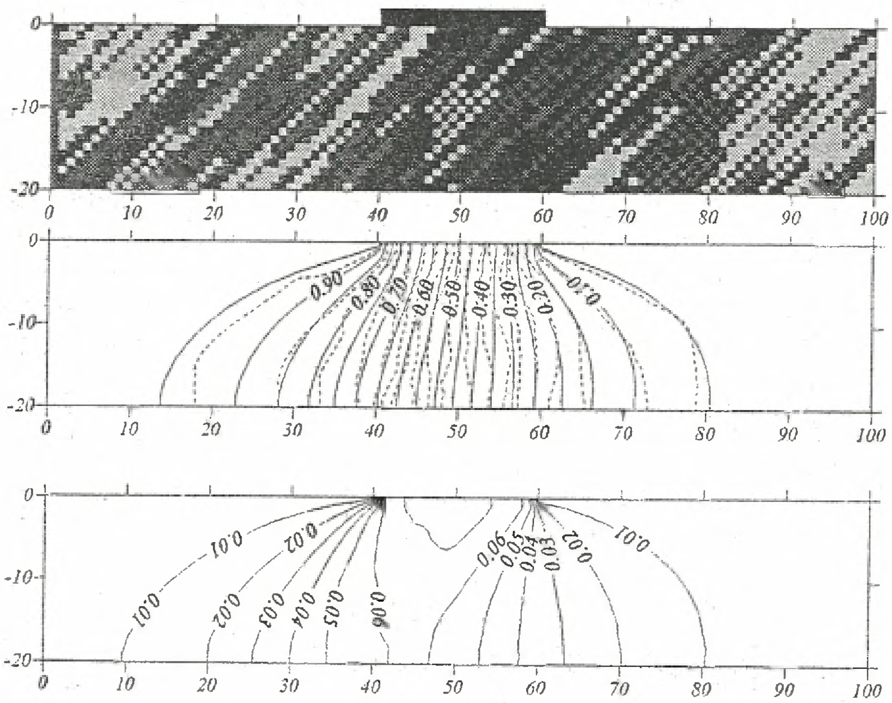

Fig.(7) Single Realisation of Negatively Inclined Bedding -45 degree (Top), Ensemble Hydraulic Head Superimposed over The Hydraulic Head of That Realisation (Middle) and Standard Deviation of The Hydraulic Head (Bottom).

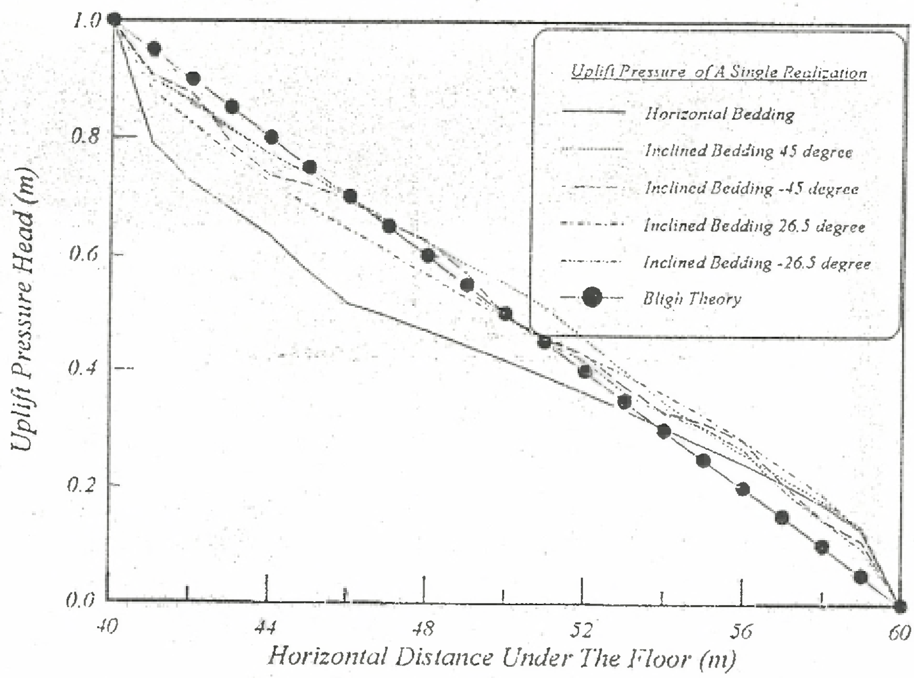

Fig. (8) Uplift Pressure of Single Realisations of The Various Geological Bedding.

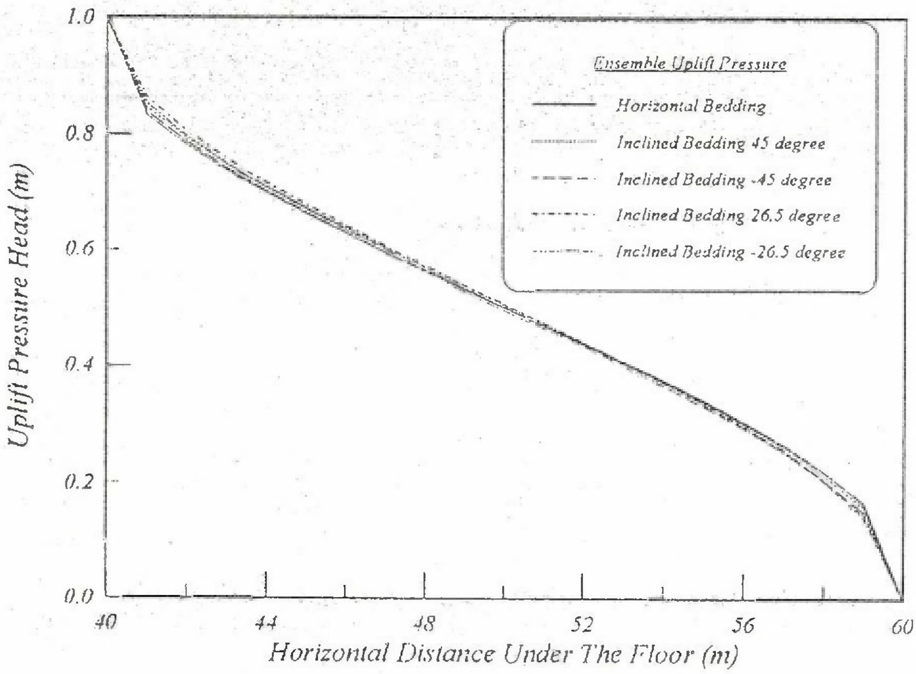

Fig. (9) Ensemble Mean of The Uplift Pressure under The Floor of The Various Geological Bedding.

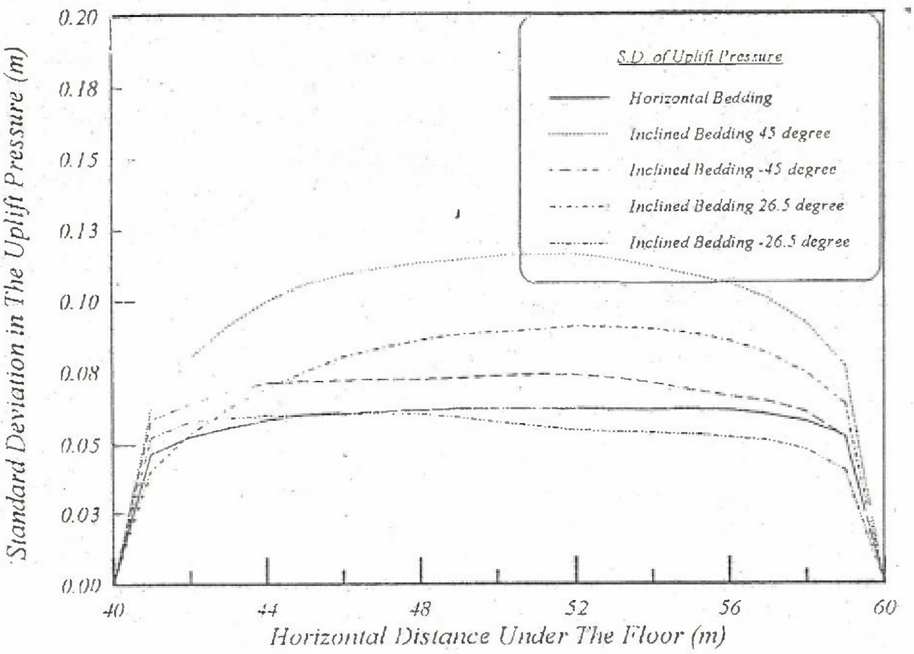

Fig. (10) Ensemble Standard Deviation (Measure of The Uncertainty) of The Uplift Pressure under The Floor of The Various Geological Bedding.

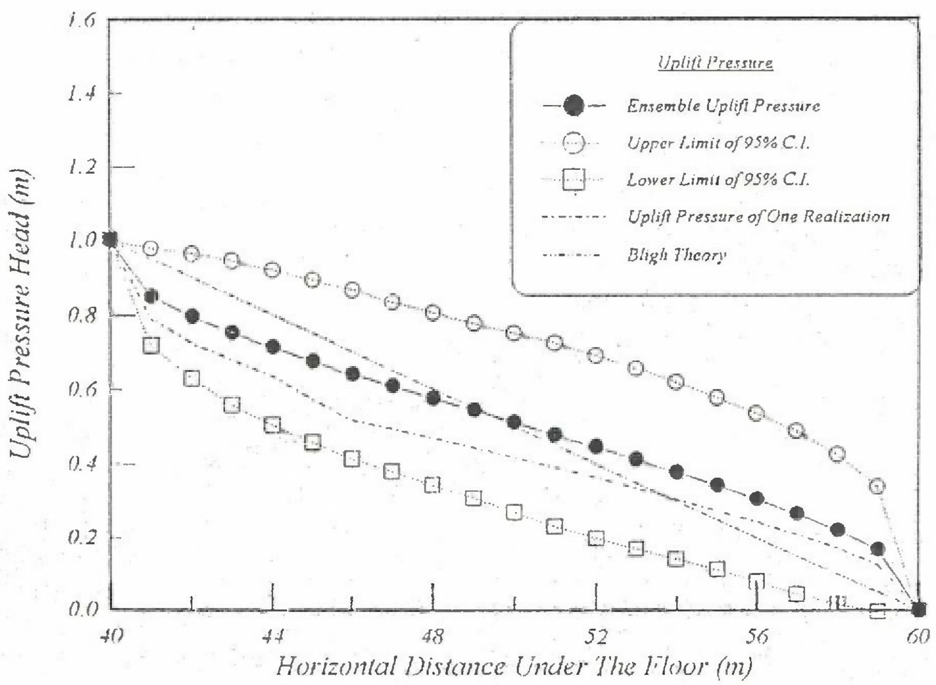

Fig. (11) Comparison of The Uplift Pressure of A Single Realisation with Its Confidence Bounds in Case of Horizontal Bedding.

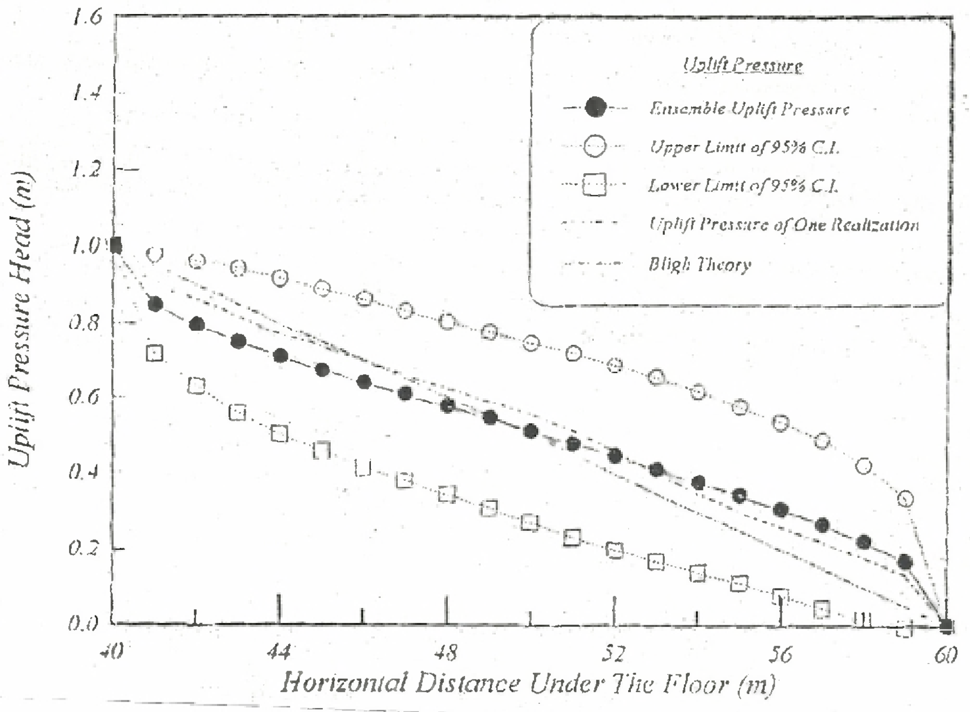

Fig. (12) Comparison of The Uplift Pressure of A Single Realisation with Its Confidence Bounds in Case of Inclined Bedding with 45 degree.

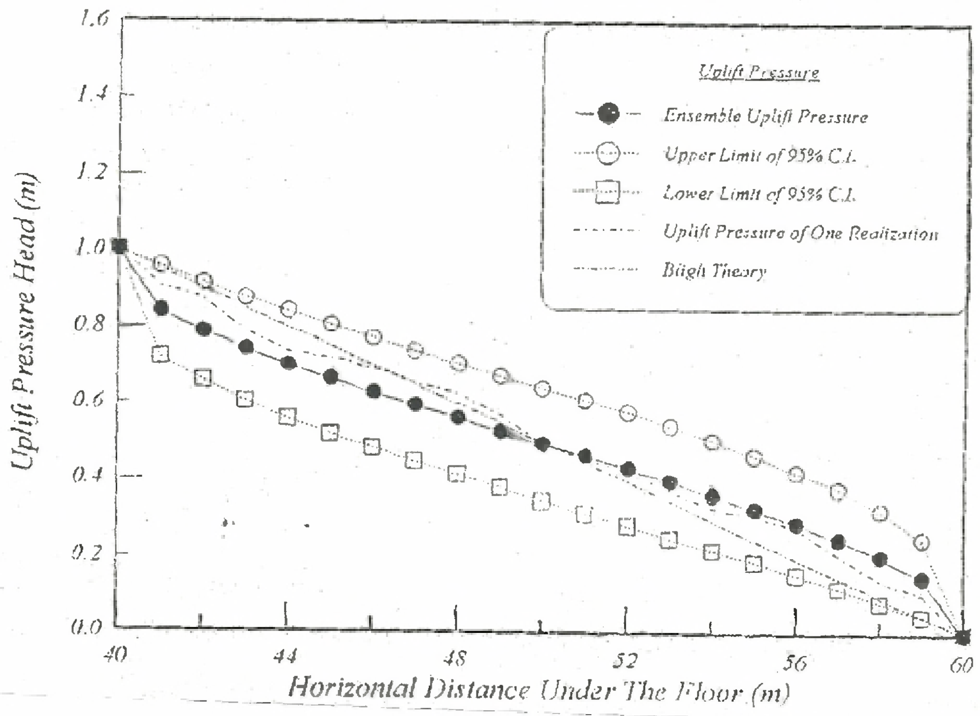

Fig. (13) Comparison of The Uplift Pressure of A Single Realisation with Its Confidence Bounds in Case of Inclined Bedding with -45 degree.

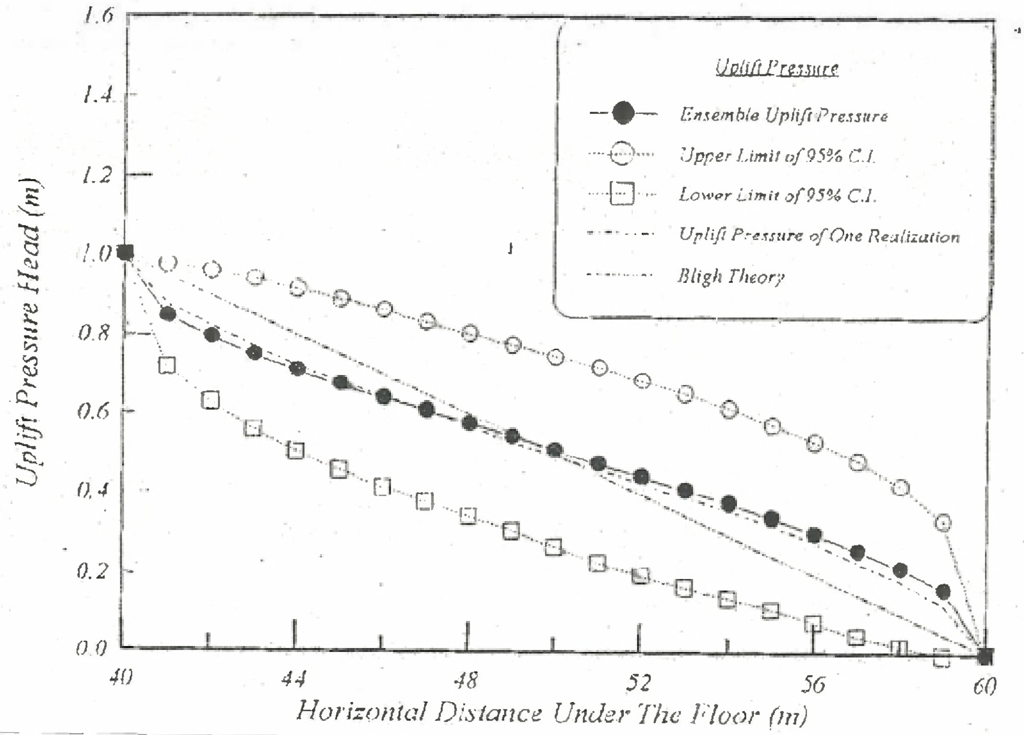

Fig. (14) Comparison of The Uplift Pressure of A Single Realisation with Its Confidence Bounds in Case of Inclined Bedding with 26.5 degree.

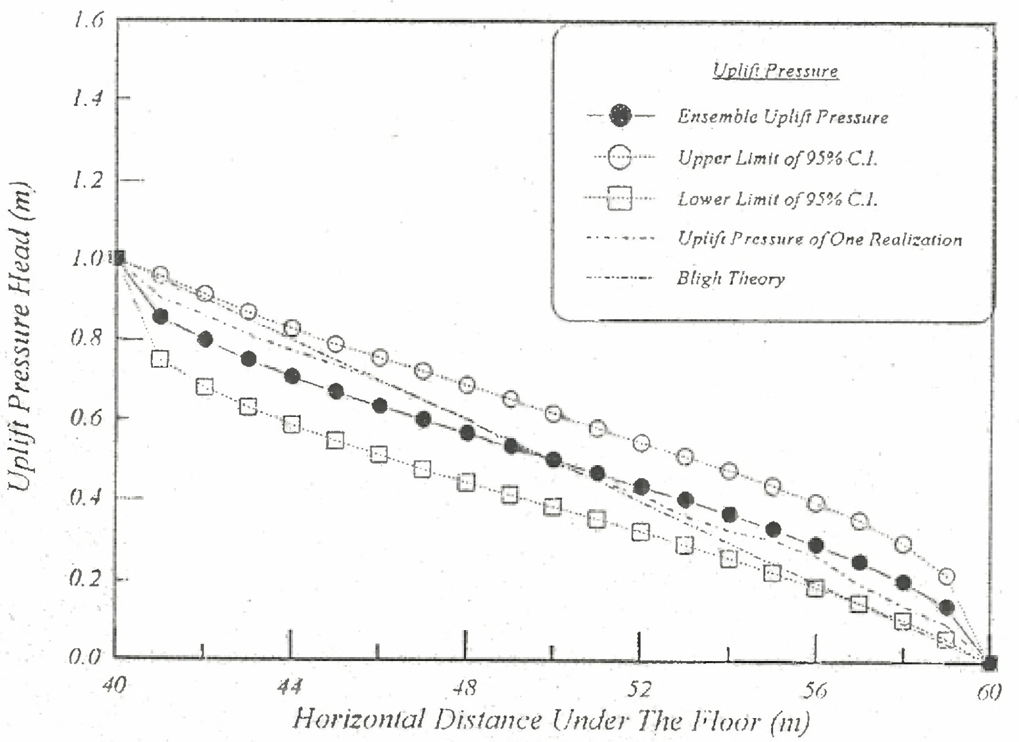

Fig. (15) Comparison of The Uplift Pressure of A Single Realisation with Its Confidence Bounds in Case of Inclined Bedding with -26.5 degree.