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Dipole Decay Rate between Conducting Plates in the Presence of a Cavity Mode

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Abstract. In the absence of external modes, cavity dimensions and dipole location are the main factors that increase, reduce and disallow dipole decay rate in the Cavity QED. The presence of external modes with short cavity length, high atom velocity and high intensity is considered as a reasonable source of constrain to the dipole vector in a fixed direction. As a result, only two orthogonal directions can be assumed: π polarization and σ polarization. These directions can be used to illustrate the variations of Cavity QED effects with dipole adjustment. Thus the distribution of the dipole decay rate across the plates separation is independent of the excited cavity mode. In contrast, during the passage of a slow atom between the plates, the dipole can adjust itself along a local electric field direction. In this work, it has been proven that the average dipole adjustment is determined at every point parallel to the direction of the electric mode vector of the excited cavity mode. Also the subordination was shown to lead to observable changes in the dipole decay rate. The above cases have been explored for situations involving sodium atoms between conducting plates of the sub-wavelength range.

1. Introduction

Modern studies on the subject of cavity quantum electrodynamics (CQED), namely those concerned with the influence of field confinement in microcavities on the properties of quantum systems, have been encompassed in planar[1-5] and cylindrical structures[6-10]. These studies have enumerated the modes and used them to evaluate the basic process of dipole decay rate distribution, within a normal cross section, between two parallel plates or within a normal cross-section of a cylinder. The outcome of these studies shows that the electric dipole moment vector of the atom is polarized in the π or σ direction. This appears to follow the traditional pattern in works in which cavity influences were investigated near a half-space[11] or between two parallel plates[1-5].

The CQED effects inside planar and cylindrical structures have more currently ordered a specific significance where the stress begins to move to the motion of atoms in hollow structures[12-15]. Such structures act as guides to both atoms and electromagnetic radiation, with the main interest being the control of atomic motion in the structure.

For a short cavity length, high atom velocity and high intensity, it is reasonable to follow traditional patterns when considering that the electric dipole vector is constrained to remain in either π or σ polarization. However, slow atom transit time can indeed be longer than a typical relaxation time. In which case, the electric dipole moment vector is forced by the cavity mode to lie along the electric mode vector at the instant position of the dipole. An adiabatic representation emerges in which the electric dipole of a moving atom continually adjusts its direction along the electric mode vector of the excited cavity mode. Its instant features are consistent with the dipole adjustment at that point. By supplying a picture of the orienting of the dipole adjustment, it can be seen that, this dynamic mode-dipole adjustment controls the whole CQED effect that arises in the absence or presence of the cavity mode.

The primary aim of this article is to investigate the effects of mode-dipole adjustment pictures on the above quantum process between two perfect conducting parallel plate structures. This system acts to totally confine all fields introduced into the vacuum region. Because of its relative simplicity as a confining structure, it has a distinguished history as a testing ground for the confinement effects in quantum electrodynamics. CQED between conducting plates have already been discussed[1-5] and the atomic motion within it for the high-speed atom case has been examined[16]. However, it appears that the effects of atomic motion on a dipole decay rate in such a fundamental system have not as yet been investigated.

In this article, two different physical situations of the dipole adjustment are considered; firstly when the distribution of the dipole decay rate between the plates is independent of the excited cavity mode. And secondly when the excited cavity mode influences the distribution of dipole adjustment, which changes the dipole decay rate. The former will be referred to as the traditional pattern, and latter as the new pattern. In section 2 expansions of the dipole decay rate in the absence of a cavity mode, following the traditional pattern, in terms of Γ_π for a dipole fixed parallel to the system axis, and Γ_σ for a dipole fixed perpendicularly to it will be demonstrated. The effects of the presence of a cavity mode on the dipole decay rate are considered in section 3. Section 4 contains conclusions and further comments.

2. Dipole Decay Rate in the Traditional Pattern

Electromagnetic modes supported by a pair of perfectly conducting, plane-parallel and infinite metallic systems are known[17]. The electric dipole located within such a structure can only couple to a set of discrete modes (or allowed modes) with cut-off frequencies for a given longitudinal wave-vector $k_{||}$ depending on the cavity dimensions. For such a structure system of separation plates L, the dispersion relation of the modes can be written as

$$k_{||} = \left\{ \left(\frac{\omega}{c}\right)^2 - \left(\frac{n\pi}{L}\right)^2 \right\}^{1/2} \quad ; \quad n = 0,1,2,3,... \tag{1}$$

The electromagnetic modes can be quantised straightforwardly and the quantisation procedure for this system is adequately described in Ref.[16].

In the absence of any external mode, the atom interacts with the vacuum modes constrained by the structure, leading to two types of physical influence. Firstly, the dipole decay rate of the atom is modified, hence becoming position-dependent. Secondly, the atom experiences energy shifts to both levels[18-20]. The modification of the dipole decay rate for an electric dipole moment $\boldsymbol{\mu}$ situated at an arbitrary point $\mathbf{r} = (\mathbf{r}_{||}, Z)$ between conducting plates is evaluated by Hinds[1]. The π polarization case is

$$\Gamma_{\pi}(Z) = \Gamma_o \sum_{n=0}^{[n_{\max}]} \frac{3\lambda}{4L}\left[1 + \left(\frac{n\lambda}{2L}\right)^2\right] \sin^2\left(\frac{n\pi}{L} Z\right) \tag{2}$$

And the σ polarization case

$$\Gamma_{\sigma}(Z) = \Gamma_o \left\{ \frac{3\lambda}{4L} + \sum_{n=1}^{[n_{\max}]} \frac{3\lambda}{2L}\left[1 - \left(\frac{n\lambda}{2L}\right)^2\right] \cos^2\left(\frac{n\pi}{L} Z\right) \right\} \tag{3}$$

where Γ_0 is the corresponding spontaneous rate in free space

$$\Gamma_0 = \frac{\mu^2 \omega_o^3}{3\pi \ \hbar \varepsilon_0 c^3} \tag{4}$$

$\lambda = 2\pi c/\omega_0$ is the free space wavelength of the dipole transition and the maximum $n_{\max}$ that can be allowed for a given separation is determined by the dispersion relation as follow

$$n_{\max} = Int\left(\frac{\omega_0 L}{c\pi}\right) = Int\left(\frac{2L}{\lambda}\right) \tag{5}$$

where $Int(\ )$ stands for the integer part of the bracketed quantity.

Focus is on the two conceivable cases of separate adjustment by assuming that the electric dipole is oriented in an unchanging direction: π polarization or σ polarization for typical cases involving sodium atoms between a pair of perfectly conducting, plane-parallel and infinite metallic system of $L = 500\ nm$ and focuses on its $3^2 s_{1/2} \leftrightarrow 3^2 p_{3/2}$ transition ($\lambda = 589\ nm$). The magnitude of the dipole matrix element associated with this transition is about $\langle d \rangle = 2.6\ ea_B$, which is consistent with the measured free space lifetime of $t \approx 16.3\ ns$ (or $\Gamma_0 = 6.13 \times 10^7\ s^{-1}$).

Figs. 1 and 2 show the variation of the dipole decay rate across the plates when the dipole is adjusted in π polarization, the dipole decay rate is zero if the dipole is very close to the plate, as shown in Fig. 1. However, when the dipole is adjusted in σ polarization, it has twice the free space value, as shown in Fig. 2. It is worth noting that, firstly, because of the $L = 500\ nm$ selected for instance objective in the previous two figures the dipole decay rate distributions arise from at most just two modes. Secondly, the contribution of each mode to the decay rate depends on the position of the atom in the spatial distribution of the field in that mode.

3. Dipole Decay Rate in the New Pattern

So far the assumption has been made that the electric dipole adjustment does not change in the course of atom motion. This is quite reasonable provided that the electric dipole vector is constrained to remain in a fixed direction by some external means, or that the time of flight through the length of the cavity is too short for the direction of the dipole vector to adjust to the local mode surroundings.

The case in which the individual electric dipoles respond to the structure mode by oscillating along the direction of the local mode will now be discussed. This physically reasonable 'mode-dipole adjustment picture' has significant results and observable changes in the dipole decay rate. Assuming that the TM_1 mode is excited, it is not difficult to derive the local angle of adjustment for the electric mode vector at all locations within a normal cross section and this local angle defines the adjustment of the electric dipole and, so, controls local decay rate.

In the view of Ref.[16], the quantization electric field function of a TM_1 mode is given by

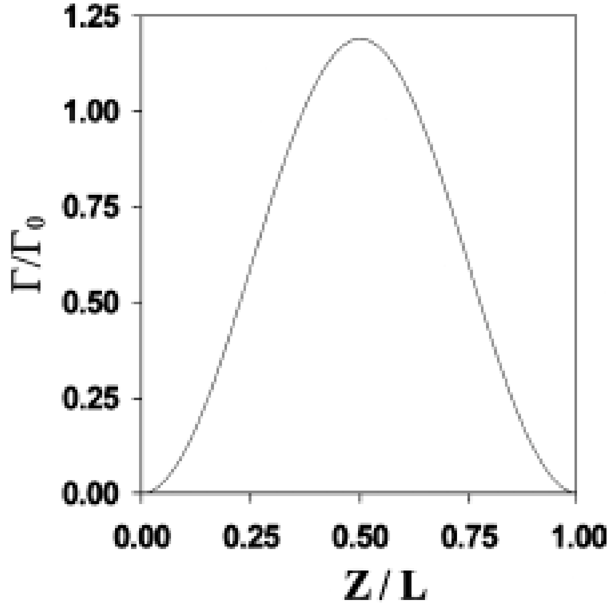

FIG. 1. Distribution plots for the dipole decay rate for an atom between a pair of perfectly conducting, parallel plates system. This plot shows the variations Γ_π/Γ_0 with the position of the atom within the plates separation.

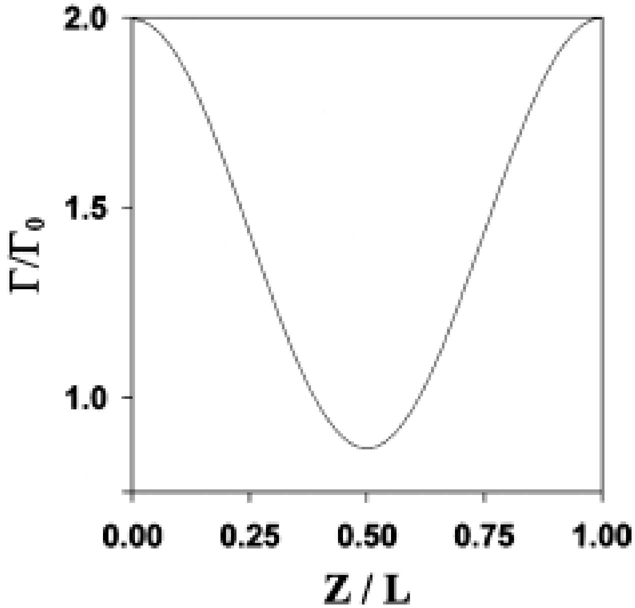

FIG. 2. Distribution plots for the dipole decay rate for an atom between a pair of perfectly conducting, parallel plates system. This plot shows the variations Γ_σ/Γ_0 with the position of the atom within the plates separation.

$$\tilde{\mathbf{E}}_p(\mathbf{k}_{||},1,\mathbf{r},t)=\left(\frac{c^2\hbar}{V\varepsilon_o\omega(k_{||},1)}\right)^{1/2}\left(\hat{\mathbf{k}}_{||}(\frac{\pi}{L})\sin(\frac{\pi}{L}Z)+\hat{\mathbf{Z}}ik_{||}\ \cos(\frac{\pi}{L}Z)\right)e^{-i(\omega(k_{||},1)t-\mathbf{k}_{||}\cdot\mathbf{r}_{||})} \quad (6)$$

where V is quantization volume. The dipole decay rate in the presence of the TM_1 mode must then be expressed as a linear combination of Γ_π and Γ_σ as determined by the local dipole adjustment

$$\Gamma(Z) = \Gamma_\pi \sin^2 \theta(Z) + \Gamma_\sigma \cos^2 \theta(Z) \quad (7)$$

where $\theta(Z)$ is the local adjustment angle of the local electric field vector at the point z

$$\theta(Z) = \tan^{-1}\left(\frac{E_\perp}{E_{||}}\right) \quad (8)$$

where $E_\perp$ is the magnitude of the perpendicular component of the electric field and $E_{||}$ is the magnitude of the longitudinal component. Therefore, the local angle can be written as

$$\theta(Z) = \tan^{-1}\left(\frac{Lk_{||}}{\pi}\ \cot(\frac{\pi}{L}Z)\right) \quad (9)$$

The distribution of the angle $\theta(Z)$ across the normal cross section is shown in Fig. 3. The schematic representation of Fig. 4 is presented to simplify and ease the understanding of the local angle shown in Fig. 3. It is clear that for this mode the angle of adjustment is 90º (in the π polarization case) at the center of the cross section and regularly reduces across the plane to zero at the plates (dipole adjusted in the σ polarization case).

In Fig. 5, the distribution for the local decay rate across the normal cross section is plotted. Evaluations are carried out for points spanning the cross section and are based on the expressions given in Eq. (7). As anticipated, it is seen that for this particular mode the local decay rate is equal to $2\Gamma_0$ at the plates which agrees with the result shown in Fig. 2 for σ polarization situated close to the plate, while the local decay rate is equal to about $1.23\Gamma_0$ at the center of the cross section which also agrees with the result found in Fig. 1 for π polarization.

These consequences come directly from the mode-dipole adjustment picture that forces the dipole to remain in the σ polarization at the plate and in the π polarization at the center. This means that the mode-dipole adjustment picture primarily governs the dipole decay rate of slow atoms. Therefore, the relevant characteristics of this decay of any system depends not only on the position and orientation of the dipole, but also on the average of the dynamic mode-dipole adjustment due to a particular excited mode.

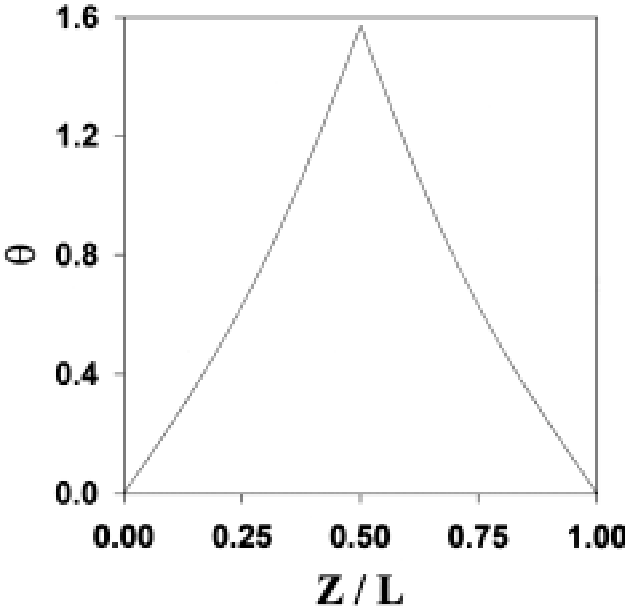

FIG. 3. Distribution plots of adjustment angle $\theta(Z)$ of electric dipole which oscillates along the local electric mode direction of an excited TM_1 mode.

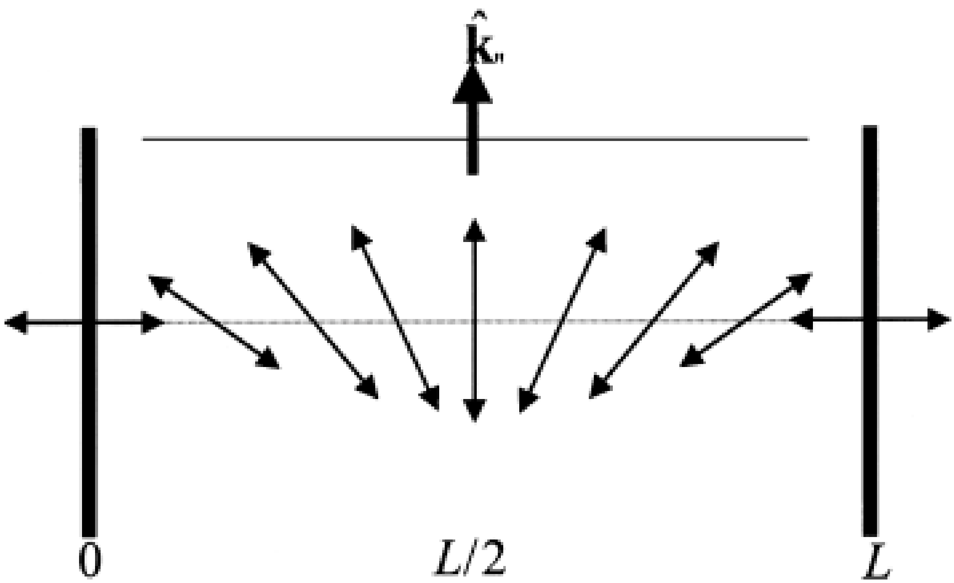

FIG. 4. Schematic representation of the dipole angles given in Fig. 3. At the center the dipole angle is 90º and it continually decreases to zero at the plate.

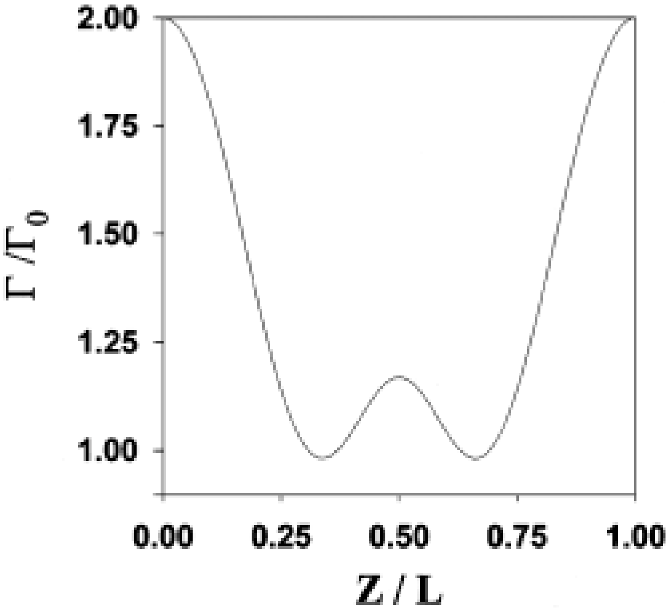

FIG. 5. Distribution plots for the dipole decay rate of adjustment angle $\theta(Z)$ of electric dipole which oscillates along the local electric mode direction of an excited Cm_1 mode for an atom between a pair of perfectly conducting, parallel plates system.

4. Comments and Conclusion

The main aim of this article is to study the role of an excited cavity mode in determining the dipole decay rate of a single atom. The analysis was conducted by studying the movement of this atom between a pair of perfectly conducting, plane-parallel and infinite metallic systems when one of its permitted modes was excited. For a very fast atom, the dipole decay rate can essentially be considered to be dependent only on a cavity dimension, the dipole location and the dipole polarization but it is independent of an excited cavity mode.

On the other hand, for a slow atom, the time of flight within the length of the system is in fact longer than a typical relaxation time. This means that the dipole vector will have enough time to adjust a every point so as to be parallel with the direction of the electric mode vector of the excited cavity mode. Therefore, in addition to the position dependent, the dipole decay rate in cavity is inherently governed by an excited cavity mode. This dependence is called the *mode-dipole adjustment picture,* which was investigated for a specific situation of a sodium atom moving between a pair of conducting plates.

As a result of the *mode-dipole adjustment picture,* the dipole decay rate at any given point between plates become identical to that of a dipole adjusted along the direction of the electric mode vector at the given point. It should be emphasized that the role of the cavity mode in the dipole decay rate is only used to determine an average direction of the electric dipole moment vector. This direction is along the direction of the electric mode vector of the excited cavity mode.

The dipole decay rate is a result of the emission and re-absorption only of the cavity vacuum modes by the dipole where the moment vector is adjusted at the given direction of the electric mode of the excited cavity mode at the point. The mode has no other role to play in the dipole decay rate in the cavity. It is important to note that the dipole decay rate is a substantial ingredient of the radiation forces responsible for channelling the atoms along the hollow region of any structure when dipolar transition frequency is appropriately tuned to an excitable mode of the structure. Therefore, the *mode-dipole adjustment picture,* in general, will control all CQED effects arising in the presence of the cavity mode such as heating, cooling and trapping of atoms[21-25].

Finally, it is worth noting that the *mode-dipole adjustment pictures* and hence the dipole decay rate distribution is dependent on the order of the mode, particularly for an excited p-polarized mode in a cylindrical system. Work along these lines is in progress and the results will be reported in due course.

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معدل انحلال ثنائي قطب موضوع بين لوحين موصلين في وجود نمط الفجوة

سعود العوفي

قسم الفيزياء ، كلية العلوم ، جامعة الملك عبد العزيز بالمدينة المنورة

المملكة العربية السعودية

المستخلص . يمكن القول وبصفة عامة أن أبعاد الفجوة وموقع ثنائي القطب تشكل العوامل الرئيسة لارتفاع أو انخفاض أو حتى منع معدل انحلال ثنائي القطب في الديناميكا الكهربائية للفجوة وذلك في حالة غياب الأنماط الخارجية . ويمكن اعتبار وجود الأنماط الخارجية وفجوة قصيرة الطول وذرات سريعة وذات شدة عالية كمصدر مناسب لإجبار متجه ثنائي القطب على اتجاه ثابت (أي جعله في اتجاه محدد). ونتيجة لذلك يمكن افتراض اتجاهين متعامدين هما استقطاب π (الاتجاه الموازي) واستقطاب σ (الاتجاه العمودي) . وهذه الاتجاهات يمكن استخدامها لتوضيح تغيرات تأثيرات الديناميكا الكهربائية الكمية للفجوة مع وضع ثنائي القطب . وبهذا التصور فإن توزيع معدل انحلال ثنائي القطب عبر مساحة المقطع لا يعتمد على نمط الفجوة المثار . وعلى النقيض من ذلك في حالة مرور ذرات ذات سرعات بطيئة فان ثنائي القطب يستطيع تعديل نفسه في اتجاه مجال كهربي محلي (النمط المثار في الفجوة) . وفي هذا البحث قمنا بحساب معدل تعديل ثنائي القطب باتجاه المجال الكهربائي وحددناه عند كل نقطة عبر مساحة المقطع . وتوصلنا إلى أن هذه التبعية تؤدي إلى تغيير ملحوظ في معدل انحلال ثنائي القطب ، حيث طبقت الحالتان أعلاه على ذرات صوديوم متحركة بين لوحين موصلين تفصلهما مسافة في مدى أقل من الطول الموجي للنمط المثار .